A novel time optimal acc-jerk limited trajectory planning for serial machining robots using NURBS curves

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This paper presents a novel acceleration-jerk limited trajectory planning for serial machining robots. First, the trajectory of machining robot is designed in the Cartesian space using the Non-uniform rational basis spline (NURBS) curve then, the position commands of the end-effector are obtained by the NURBS interpolator containing the advised time optimal acc-jerk limited feedrate profile along the path. To evaluate the accuracy of the kinematic and dynamic analyses, the contour following task is investigated using the PID computed-torque controller for a KUKA machining robot. The results confirmed that the proposed method yields satisfactory performance in the trajectory tracking control problem.

Introduction

Trajectory planning is one of the most important issues in robotics and refers to generating the position commands, velocity and acceleration for the end-effector of a robot at each sampling time [1]. An appropriate trajectory planning yields a satisfactory trajectory tracking and machining surface quality.

Despite of several existing CAD-based approaches for trajectory planning enhanced with Bezier and B-spline parametric curves, trajectory planning using NURBS curves in robotics has been limited to some studies. For instance, A procedure to build NURBS motion interpolants to avoid boundary orientation poses for PUMA was presented in [2]. Zhao et al. [3] used the second-order Taylor's expansion for the trajectory interpolation of SCARA robot with a constant feedrate based on NURBS curve. But, the robot dynamics and control motion on the NURBS trajectory were not discussed in their work. Recently, Jahanpour et al. [4] employed NURBS curves for trajectory planning of parallel machining robots. However, in their work, the joints robot kinematic constraints were not taken into account. In this paper, A novel time optimal acc-jerk limited trajectory planning for end-effector of serial machining robots using parametric the NURBS curve is proposed. Moreover, the trajectory tracking control is studied with the computed torque controller.

The Proposed Acc-Jerk Limited Trajectory Planning Scheme

The NURBS curve of degree *p*, defined by given *n*+1 control points $P_0, P_1, \ldots, P_n$ with corresponding weights $w_0, w_1, \ldots, w_n$ and the knot vector $U=\{u_0, u_0, ...u_m\}$ is:

$$C(u) = \frac{\sum_{k=0}^{n} N_{k,p}(u) w_k P_k}{\sum_{i=0}^{n} N_{i,p}(u) w_i} = \sum_{k=0}^{n} R_{k,p}(u) P_k \qquad (1)$$

Where, *u* is the curve parameter. Also $R_{k,p}(u)$ and $N_{k,p}(u)$ are the Rational B-spline and basis functions of degree *p*, respectively, and have been given in [5].

The overall structure of the proposed trajectory planning scheme in the Cartesian space has been depicted in Fig.1. According to Fig.1, At first the geometrical path planning is carried out using the parametric NURBS curve. Then the trajectory generation is performed employing the NURBS interpolation algorithm with the recommended motion planning architecture equipped with acc-jerk limited feedrate profile along the tool path. In this paper, in order to implement the trajectory generation, position commands along the tool path are computed by the NURBS interpolator using the second-order Taylor's expansion method given in [6]. Also, to obtain a smooth motion transition between different poses along the tool path by the end-effector, the acc-jerk limited feedrate profile given in [7] is employed in the NURBS interpolation procedure. Using this acc-jerk limited feedrate profile in the interpolation algorithm, the position commands for the end-effector are computed at each sampling time.

The Time Optimal Acc-Jerk Limited Trajectory Planning with Joint Kinematic Constraints

After generating the position commands along the represented NURBS curve the inverse kinematic (IK) calculation must be used to obtain the desired joint motions, i.e. joints position angle, velocity, acceleration and jerk. In this paper, an optimization method is also proposed to achieve the minimum total machining time needed to move the robot within the given joint kinematic constraints of all axis. In fact, the total machining time is defined as the objective function (*O.F.*) in the optimization algorithm, i.e. *O.F= total machining time*.

The namely feedrate value of F is determined using the optimization algorithm such that the total machining time be minimized while the all given joint kinematic constraints in the joint space are also satisfied. The pattern search algorithm is employed in this paper to achieve the optimized total machining time. The following nonlinear kinematic constraints for the velocity, acceleration and jerk values of each joint of robot are used in our optimization algorithm.

$$\begin{cases} \left|\dot{q}_i\left(t\right)\right|_{actual} - V_{i_{allow}} \le 0 \\ \left|\ddot{q}_i\left(t\right)\right|_{actual} - A_{i_{allow}} \le 0 \;\; ; \;\; i=1,...,6 \\ \left|\dddot{q}_i\left(t\right)\right|_{actual} - J_{i_{allow}} \le 0 \end{cases} \tag{2}$$

The pattern search algorithm starts with an initial value for F. At each successful iteration, the pattern search algorithm changes the namely feedrate via the nonlinear kinematic constraints as Eq.2. Finally, the minimized total machining time and the optimum variable are computed by the pattern search algorithm.

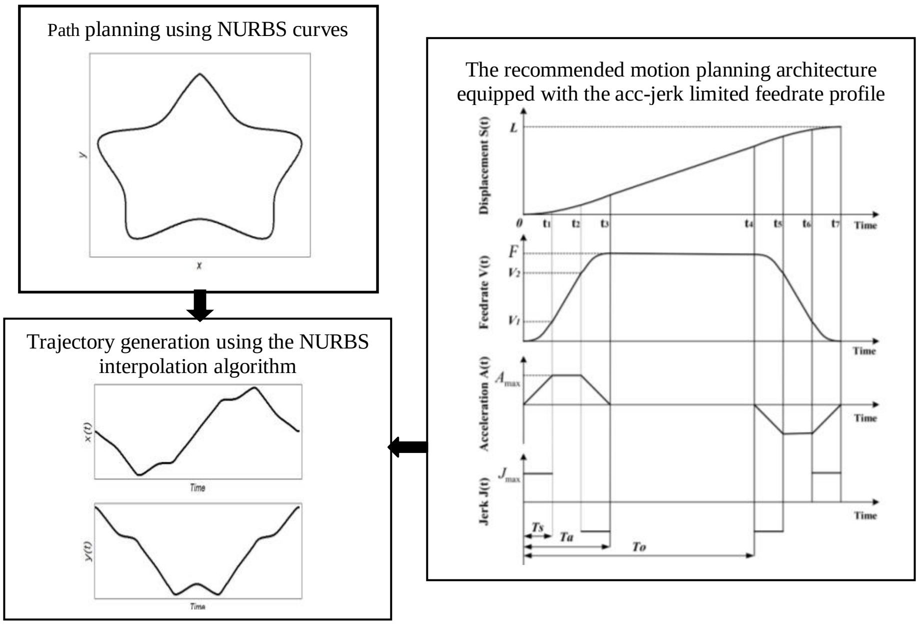


Fig. 1 Structure of the proposed trajectory planning scheme

NURBS Curve Following by the KUKA KR16 Machining Robot

In this paper, the industrial serial manipulator KUKA KR16 has been selected as case study. The closed form solution of the IK for this robot has been given in [8]. The dynamic equation of motion equipped with PID computed-torque controller for a serial manipulator is derived as [9]:

$$\tau = M\left(q\right)\left(\ddot{q}_d + K_v\dot{e} + K_p e + K_i\varepsilon\right) + N\left(q,\dot{q}\right) \tag{3}$$

Where $\dot{\varepsilon} = e = q_d(t) - q(t)$, τ is the vector of torques applied at joints and the desired joint positions, velocities, and accelerations are $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$, respectively.

In this paper, the "star"-shaped NURBS curve as an example is used to evaluate the performance of the proposed time optimal acc-jerk limited trajectory planning scheme. The parameters for constructing the "star" shaped NURBS tool path are given in the Table 1. The selected options for the pattern search algorithm are chosen according to [10] and the controller gains for the PID are chosen as $k_p = 100\ (1/\mathrm{s}^2)$, $k_v = 20\ (1/\mathrm{s})$ and $k_i = 500\ (1/\mathrm{s}^3)$. The sampling period is also set to 1ms. Moreover, the maximum values of acceleration and jerk along the path are selected as $A_m = 500\ (\mathrm{mm/s^2})$ and, $J_m = 20000\ (\mathrm{mm/s^3})$, respectively. Besides, the angular acceleration allowable limits for the first- to end-joint are chosen as 170,160, 180, 350, 350 and 650 deg/s^2 respectively and the maximum angular jerk limit for all joints is selected as 1500 deg/s^3.

The optimized feedrate obtained from the pattern search algorithm with the aforementioned given constraints for the acceleration and jerk values along the path is $F = 89.035\ (\mathrm{mm/s})$. To evaluate the performance of the controller system used for the trajectory tracking task, the starting point of the actual/controlled trajectory is considered differ from the desired reference trajectory starting point. The desired NURBS trajectory as the path and the actual trajectory are shown in Fig. 2. The joints angular acceleration and joints angular jerk are shown in Fig. 3. Although the starting point of the motion in the actual trajectory differs from the desired one (See the upper enlarged region in Fig. 2) as can be seen in Fig. 3, the actual trajectory closely matches the desired NURBS curve. It is found that the actual maximum contour error is 67.2 μm that occurs at the portion of the contour related to the lower enlarged region shown in Fig. 2. According to Fig. 3, it is observed that the maximum joints angular acceleration and the maximum joints angular jerk obtained from the IK with the recommended motion planning architecture equipped with the acc-jerk limited feedrate profile are lower than their allowable limits.

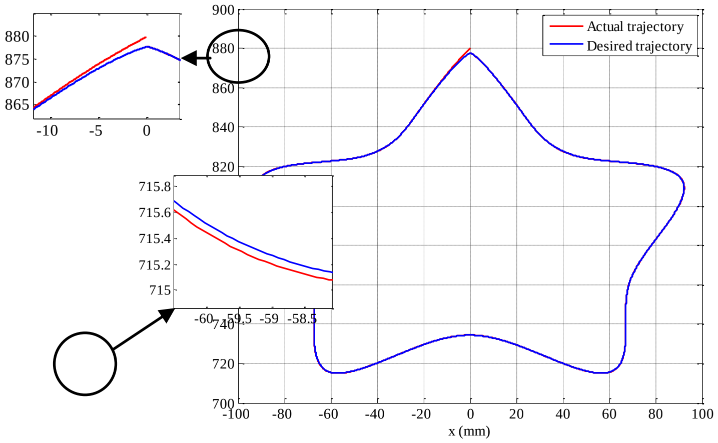
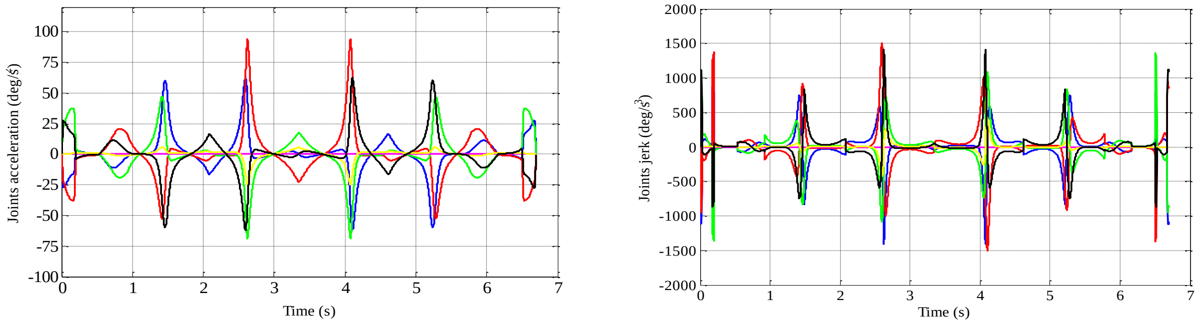


Fig. 2 The actual and desired trajectories in the contour-following task

Table 1

The parameters for constructing the “star” shaped NURBS tool path

NURBS parameters	
Degree	3
Knot vector	[0, 0, 0, 0, 0.0956, 0.1629, 0.2393, 0.3224, 0.4098, 0.5000, 0.5902, 0.6776, 0.7607, 0.8371, 0.9044, 1, 1, 1, 1]
Control points	(0, 0, 0), (-5, -3, 0), (-21, -27, 0), (-40, -60, 0), (-122, -50, 0), (-60, -115, 0), (-84, -200, 0), (0, -125, 0), (84, -200, 0), (60, -115, 0), (122, -50, 0), (40, -60, 0), (21, -27, 0), (5, -3, 0), (0, 0, 0)
Weights	[1, 1, 1, 1, 1, 2, 1, 1.5, 1, 2, 1, 1, 1, 1, 1]



(a) The joints angular acceleration

(b) The joints angular jerk

Fig. 3 The rotational characteristics of the machining KUKA KR16 robot joints: $(\ddot{\theta}_1,\dddot{\theta}_1)$, $(\ddot{\theta}_2,\dddot{\theta}_2)$, $(\ddot{\theta}_3,\dddot{\theta}_3)$, $(\ddot{\theta}_4,\dddot{\theta}_4)$, $(\ddot{\theta}_5,\dddot{\theta}_5)$, $(\ddot{\theta}_6,\dddot{\theta}_6)$

Conclusion

This paper has introduced a new time optimal acc-jerk limited CAD-based trajectory for serial machining robots. The feedrate profile for the end-effector motion has also been optimized using the pattern search algorithm equipped with the nonlinear constraints of velocity, acceleration and jerk of all joints to minimize the total machining time. The “star” contour following task results demonstrated that the proposed time optimal acc-jerk limited trajectory planning for the KUKA KR16 along with the designed PID computed-torque controller are capable of maintaining the tracking accuracy within a tight tolerance limit while all joint kinematic constraints are satisfied.

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